VARIABLE THERMAL CONDITIONS FOR FLOODED ROOFS

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A solution is offered for the problem of unsteady heat conduction in a two-layer system consisting of a layer of water and a solid base of finite thickness with variable heat sources at the interface. The thermal conditions at each surface are similar to those for flooded roofs.

Flooded roofs provide effective thermal insulation, but the lack of methods of performing the corresponding calculations is one of the reasons why they are still not being widely used. The development of a suitable analytic method for summer conditions would allow disconnected observations to be generalized, and an engineering design method to be perfected.

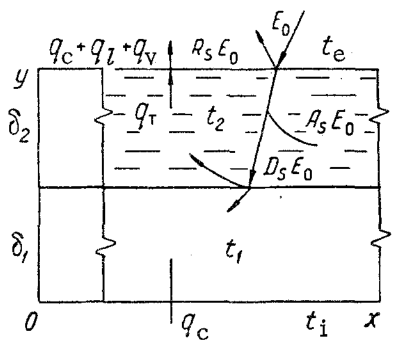

Fig. System of heat fluxes assumed for analysis of flooded roofs.

Consider a section of a flooded roof, far enough from the eaves for heat flow in the horizontal direction to be neglected, so that only vertical heat transfer in the direction of the y axis (see Fig.) need be considered.

The water surface receives direct and scattered solar radiation whose intensity varies with time, $E_0(\tau) = E_0 + E_1 \exp i\omega(\tau)$. One part of the direct and scattered radiation $R_S E_O(\tau)$ is reflected from the surface, a second part $A_S E_O(\tau)$ is absorbed by the water, while a third part $D_S E_O(\tau)$ passes through the layer of water and reaches the surface of the roof.

According to data of the Solar Engineering Commission, the scattered radiation on a clear day, this being the most unfavorable with regard to the heating of building surfaces, is about 10% of the direct radiation. The spectra of direct and scattered solar radiation differ only slightly, so the coefficients of reflection, absorption, and transmission of a layer of water for solar radiation (R_S, A_S, D_S) can be considered the same for radiation of both kinds. These coefficients vary during a 24-hr period and depend on the altitude of the sun. For simplicity of analysis, however, we shall assume the values of A_s and D_s to be constant and equal to their mean values during the hours of maximum solar radiation (9 to 15 hr), and the value of R_s to be zero, which gives somewhat exaggerated values of the heat flux falling on the roof.

Our investigations to determine the transmission coefficient of a layer of water for solar radiation have shown that for practical purposes the following empirical formula may be used (for latitudes from 30° to 60° and water layer thicknesses 0.02 to 0.1 m): $D_s = 0.75$ to $2.5\,\delta_2$. The solar radiation absorbed by the water $A_S E_O(\tau)$ is distributed uniformly over the whole volume. This part of the radiation acts like a uniformly distributed internal heat source.

The fraction $D_S E_O(\tau)$ of the solar radiation is converted into heat energy upon reaching the surface of the roof. We shall therefore regard this surface as having uniformly distributed heat sources. Part of this heat goes to heat the water, while the rest goes to heat the roof.

The location of the heated surface under the water layer leads to circulation of the water and convective heat transfer. It is conventional to regard a complex heat transfer process such as this in fluid layers as an elementary phenomenon of heat exchange due to conduction. We shall therefore introduce the concept of equivalent thermal conductivity, or the thermal conductivity of a solid transmitting a heat flux of the same density as the flux passing through the liquid layer due to convective heat transfer for the same temperature difference [2].

Using this concept, we may apply the following heat conduction equation to the water layer:

$$\frac{\partial t_2}{\partial \tau} = a_2 \left[\frac{\partial^2 t_2}{\partial y^2} + \frac{A_s}{\lambda_2 \delta_2} E_0(\tau) \right]. \tag{1}$$

The second term on the right side of (1) is the internal heat source, uniformly distributed over the volume.

The roof is homogeneous, plane-parallel, and, as we have stipulated, infinite, so that the distribution of the heat sources will obey the homogeneous one-dimensional equation of heat conduction

$$\frac{\partial t_1}{\partial \tau} = a_1 \frac{\partial^2 t_1}{\partial y^2}. \tag{2}$$

Let us examine the heat transfer conditions at the water surface. We may write the following equation of thermal equilibrium at any moment of time:

$$q_T - q_C - q_l - q_V = 0. \tag{3}$$

Using the concept of equivalent thermal conductivity, we can determine the amount of heat supplied to the surface of the water due to its thermal conductivity:

$$q_T = -\lambda_2 \frac{\partial t_2}{\partial y}. \tag{4}$$

The amount of heat released by the water surface to the outside air through convection is given by the equation:

$$q_C = \alpha_e^c (t_2 - t_e). \tag{5}$$

Since the calculations are done for the worst conditions of heat transfer from the roof to the surrounding air, i. e., for the lowest velocities of the outside air, and for the hottest 5-day period of the hottest month, when the 24-hr variation of outside air temperature is small, in (5) we can make the approximation $t_e = t_e^m$.

Moreover, evaluation of the terms of the heat balance equation (3) has shown [3] that the amount of heat lost due to evaporation is considerably more than the amount released due to convection, and the assumption $t_e = t_e^m = \text{const}$ does not introduce much of an error into the final results.

The heat loss from the surface in the form of long-wave radiation to the sky is given by the expression:

$$q_l = \alpha_e^r (t_2 - t_0). \tag{6}$$

According to the data of [5], we may take $t_0 = t_e^m - 18°C$.

The amount of heat lost by the water in the form of evaporation is determined by means of the Dal'tok [1] formula:

$$q_V = (0.0178 + 0.0152v) \frac{760}{b} (p_s - \varphi p_{ss}) r. \tag{7}$$

It follows from (7) that the greater φ, the less heat is lost by evaporation and the more strongly the roof is heated. The greatest relative humidity of the air usually occurs on a dull rainy day, when the air temperature is low, and the flux of solar radiation is low. For buildings with flooded roofs, clear sunny days are the most unfavorable from the point of view of heating, particularly 4-5 of them in succession. The absolute humidity on such days at places remote from the great basins remains approximately constant and is determined by the lowest 24-hr temperature of the outside air t_e^{min}. Assuming $\varphi = 100\%$ for the period of minimum 24-hr air temperature, we obtain a partial vapor pressure $\varphi p_{ss} = p_{ss}^{min}$, which stays approximately constant during the 24 hours. Then

$$q_V = (0.0178 + 0.0152v) \frac{760}{b} (p_s - p_{ss}^{min}) r. \tag{7'}$$

Observations show [6] that the range of variation of the temperature of a water surface relative to the mean 24-hr outside air temperature t_e^m is 10-20°C during 24 hours.

Making the approximation that in the limits from $t_2' = 10°C$ to $t_2'' = 40°C$ the curve $p_s = f(t_2)$ is a straight line, we obtain

$$p_s = p_s' + \frac{p_s'' - p_s'}{t_2'' - t_2'} (t_2 - t_2') = p_s' + 1.47 (t_2 - t_2'). \tag{8}$$

The range of temperature chosen, 10-40°C, does not mean that the water temperature cannot be above 40°C nor below 10°C, but only presumes that for most of the time the temperature of the water surface will fall within these limits.

Observations show [6] that the minimum air temperature does not differ much from the minimum temperature of the water surface (1-4°C), and, since the relative humidity of the air is close to 100% at the minimum air temperature, we shall assume that $p_{ss}^{min} \cong p_s'$, and $t_2 \cong t_e^{min}$. Then, taking (8) into account, (7') may be written:

$$q_V = \alpha_e^V (t_2 - t_e^{min}). \tag{7''}$$

Here $\alpha_e^V = 15.2 + 13v$.

Taking into account (4)-(6) and (7'), the heat balance equation at the water surface may be written as follows:

$$t_2 - \frac{\alpha_e^V t_e^{min} + \alpha_e^r t_0 + \alpha_e^c t_e^m}{\alpha_e^V + \alpha_e^r + \alpha_e^c} + \frac{\lambda_2}{\alpha_e^V + \alpha_e^c + \alpha_e^r} \frac{\partial t_2}{\partial y} = 0. \tag{3'}$$

Equation (3') is one of the boundary conditions (at the boundary $y = \delta_1 + \delta_2$) for solving (1) and (2).

The other boundary condition (plane $y = 0$) is the convective heat transfer between the inside surface of the roof and the inside air, the temperature of which is constant and equal to t_i:

$$-\lambda_1 \frac{\partial t_1}{\partial y} = \alpha_1 (t_1 - t_i). \tag{9}$$

The interface conditions ($y = \delta_1$) may be written as:

$$\lambda_2 \frac{\partial t_2}{\partial y} + D_s E_0(\tau) = \lambda_1 \frac{\partial t_1}{\partial y}, \tag{10}$$

$$t_1 = t_2. \tag{11}$$

In the case of solar irradiation of a flooded roof we are concerned with multiple periodic thermal influences (4-th or 5-th hot day after beginning of sunny weather) that have acquired a steady character.

This results in the establishment in the roof of a steady periodic condition in which the temperature at any point of the roof describes a harmonic oscillation with amplitude varying with the roof thickness.

In these circumstances the initial temperature distribution does not have an appreciable influence on the temperature field of the roof. Therefore the problem is solved without initial conditions.

The system of equations (1) and (2), with boundary conditions (3'), (9)-(11), has been solved by the method of separation of variables [4].

The temperature field of the solid base of the roof is

$$t_1 = \frac{\alpha_2 \lambda_2 (\theta - t_i) + E_0(\lambda_2 + D_s \alpha_2 \delta_2)}{\lambda_1 \lambda_2 (\alpha_1 + \alpha_2) + \alpha_1 \alpha_2 (\delta_1 \lambda_1 + \delta_2 \lambda_2)} (\alpha_1 y + \lambda_1) +$$
$$+ [A_1 \operatorname{sh}(\sqrt{i\omega/a_1}\, y) + B_1 \operatorname{ch}(\sqrt{i\omega/a_1}\, y)] \exp i\omega\tau + t_i.$$

The specific heat flux in the base is

$$q_1 = \alpha_1 \lambda_1 \frac{\alpha_2 \lambda_2 (t_i - \theta) - E_0(\lambda_2 + D_s \alpha_2 \delta_2)}{\lambda_1 \lambda_2 (\alpha_1 + \alpha_2) + \alpha_1 \alpha_2 (\delta_1 \lambda_2 + \delta_2 \lambda_1)} -$$
$$- \lambda_1 \sqrt{i\omega/a_1} [A_1 \operatorname{ch}(\sqrt{i\omega/a_1}\, y) + B_1 \operatorname{sh}(\sqrt{i\omega/a_1}\, y)] \exp i\omega\tau.$$

where

$$\theta = \frac{\alpha_e^v t_e^{min} + \alpha_e^r t_0 + \alpha_e^c t_e^m}{\alpha_e^v + \alpha_e^r + \alpha_e^c};$$

$$A_1 = \{2a_2 \alpha_1 \alpha_2 (1 - D_s) E_1 [(\lambda_1 \sqrt{\omega/a_1} + \alpha_1) + \alpha_1 i] \exp[-\sqrt{i\omega/a_2}(2\delta_1 + \delta_2) -$$
$$- \sqrt{i\omega/a_1}\,\delta_1]\} \{\lambda_2 \delta_2 \omega \sqrt{\omega} (\lambda_2/\sqrt{a_2} - \lambda_1/\sqrt{a_1})(\lambda_1^2 \omega/2a_1 + \lambda_1 \sqrt{\omega/a_1}\alpha_1 + \alpha_1^2)\}^{-1};$$

$$B_1 = \frac{4a_2 \alpha_2 (1 - D_s) \lambda_1 E_1 \exp[-\sqrt{i\omega/a_2}(2\delta_1 + \delta_2) - \sqrt{i\omega/a_1}\,\delta_1]}{\omega \lambda_2 \delta_2 (\lambda_2 \sqrt{a_1/a_2} - \lambda_1) \left[\frac{\lambda_1}{2}\sqrt{\omega/a_1} - \left(\frac{\lambda_1}{2}\sqrt{\omega/a_1} + \alpha_1\right) i\right]}.$$

NOTATION

a_1, a_2 – thermal diffusivity of roof material and water; b – barometric pressure; E_0 – total irradiation intensity on a horizontal surface due to direct and scattered solar radiation; p_s and p_e – vapor pressure at water surface at temperature of evaporating liquid and in outside air; p_{ss} and p_s' – saturation pressure at the air temperature, and the pressure corresponding to minimum and maximum water surface temperature for the chosen temperature range $t_2' - t_2''$; q – specific heat flux; q_t, q_c, q_l, and q_v – the amount of heat supplied to the surface of the water due to the thermal conductivity of the water, reflected to the surrounding air by convection, and lost to the surrounding medium as long-wave radiation and by evaporation; r – heat of vaporization of water; t – temperature; t_0 – reference temperature of sky; v – velocity of air; t_e – temperature of outside air; t_e^m – 24-hr mean temperature of outside air; α_e – heat transfer coefficient at water surface; α_e^r and α_e^c – coefficients of radiant and convective heat transfer between water surface and outside air; α_e^v – conditional heat transfer coefficient taking into account evaporation of water; α_i – heat transfer coefficient at inside surface of roof; δ_2 and δ_1 — thickness of water layer and roof; λ_1 and λ_2 — thermal conductivity of roof and of water; ω — angular frequency of oscillation; τ — time; φ — relative humidity.

REFERENCES

1. A. V. Lykov, Kinetics and Dynamics of Drying and Wetting Processes [in Russian], Gizlegprom, 1938.
2. M. A. Mikheev, Fundamentals of Heat Transfer [in Russian], Gosenergoizdat, 1956.
3. A. V. Nesterenko, ZhTF, no. 4, 1954.
4. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Gostekhizdat, 1953.
5. A. M. Shchklover, B. F. Vasil'ev, and F. V. Ushkov, Fundamentals of Building Heat Physics [in Russian], Gosstroiizdat, 1956.
6. F. C. Houghton, H. T. Olson, and C. Gutberlet, Heating, Piping, and Air Conditioning, 12, no. 7, 1940.

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